A comprehensive modeling framework for workmen in field of industry with cooperative Game theory approach.

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➢ Abstract

Stutdown-shartup activity planner’s main goal is to complete project with minimal time as well as minimal cost. Time, being one of most vital factors during process startup of any industry, can be traded between various divisions like production and technical, mechanical, electrical or human resource within the industry in sequential projects. Much effort (e.g. resources which demand cost) is required to complete the activities in shorter time. There is a trade-off relationship between activity time duration and its resources. In optimal case for trading time, various divisions have reasonable incentive to cooperate. In this paper, we worked for the problem in field of industry cooperating and then we propose a model to solve it based on games theory (*n*-person cooperative games). Finally, a case study is represented to more comprehensively illustrate the problem. Results from utilization of the proposed model show that while optimizing total cost, all departments can negotiate to fairly distribute benefits from cooperation in core space. The Shapley value as well as the concepts of nucleolus can be suggested.

Keywords: *Game theory, project management, employee, cost allocation*

➢ Literature and model review

Application of game theory in industry is still in the beginning of its practical applications though, when decision makers are assumed to have rational behavior, the game theory may be efficiently employed to analyze conditions for the best beneficial decisions. Most of the modeling applications of game theory in construction management, although limited in number, have employed non-cooperative static and dynamic environment to resolve industrial crisis.

Time-cost trade-off analysis is one of the most important aspects of construction project planning and control. There are trade-offs between time and cost to complete the activities of a project; in general, the less expensive the resources used, the longer it takes to complete an activity. Using critical path method (CPM), the overall project cost can be reduced by using less expensive resources for noncritical activities without impacting the project duration. Existing methods for time-cost trade-off analysis focus on using heuristics or mathematical programming. These methods, however, are not efficient enough to solve large-scale CPM networks (hundreds of activities or more). Analogous to natural selection and genetics in reproduction, genetic algorithms (GAs) have been successfully adopted to solve many science and engineering problems and have proven to be an efficient means for searching optimal solutions in a large problem domain. Feng [2] proposed an algorithm based on the principles of GAs for construction time-cost trade-off optimization. In other work Feng [3], Simulation techniques are useful for analyzing stochastic effects, but a general strategy/algorithm is needed to guide the analysis to obtain optimal solutions.

Ho [7] studied the impacts of bid compensation and to develop appropriate bid compensation strategies to analyze the behavioral dynamics between competing bidders and project owners. A bid compensation model based on game theoretic analysis is developed during this study. The model provides equilibrium solutions under bid compensation, quantitative formula, and qualitative implications for the formation of bid compensation strategies. Further Ho [8] provided theoretic foundations to policy makers for prescribing effective public-private partnership procurement and management policies, and for examining the quality policies. This study offered researchers a framework and a methodology to understand the behavioral dynamics. In recent Ho [9] analyzed through a game framework the behaviour of the players when confronted with opposite objectives in the allocation of risks. This model shows that when guarantees have a higher value than financial loss we are confronted with strategic behaviour and potential moral hazard problems. Medda [10] proposed the model in which Formwork subcontractors that hire open shop workers, rather than union workers can win more contracts and earn more profits from general contractors because of greater agility and lower costs. According to him a subcontractor may earn even more profit if it collaborates with others in a coalition. Payoff functions for individual subcontractors and a group of subcontractors in a coalition are formulated. Profit can also be reasonably allocated to each subcontractor in a coalition using the Shapley value and nucleolus. Shen [14] extended research work in which the build-operate-transfer (BOT) concession model (BOTCcM) to a new method for identifying a concession period by using bargaining-game theory to enable the identification of a specific concession period, which takes into account the bargaining behavior of the two parties concerned in engaging a BOT contract, namely, the investor and the government concerned. Perng [11] studied that formwork subcontractors that hire open shop workers, rather than union workers can win more contracts and earn more profits from general contractors because of greater agility and lower costs. A subcontractor may earn even more profit if it collaborates with others in a coalition. Payoff functions for individual subcontractors and a group of subcontractors in a coalition are formulated. He analyses that the profit can also be reasonably allocated to each subcontractor in a coalition using the Shapley value and nucleolus. Asgari [1] proposed an idea that time can be traded between subcontractors in sequential projects and save the cost without any restriction based on primary contracts signed with general contractor. Thus, various optimization approaches have been used to solve the construction scheduling problem, and they can be classified as mathematical, heuristic and metaheuristic methods. Literatures revels that few authors used a hybrid of linear programming and integer programming. Hegazy [6] used genetic algorithm, while few authors used a mixture of machine learning and genetic algorithm. All these models targeted minimizing the cost without due attention to reduce the time simultaneously.

In this paper we have extended cooperative game modeling for permanent employee cum different subcontractor's casual labours collaboration on time trading during process startup activities. Loosely speaking, process startup activities are totally controlled by planning department and they have power over all company employee and hired subcontractor’s labours. Time, being a most key factor in successful process startup activities, can be traded in sequential projects. First, a cooperative game model is developed for efficient and beneficial use of available time in sequential projects with company permanent employees and different subcontractors and its performance is illustrated with a case example.

➢ 2. Cooperative game theory and solution concepts

In game theory [5], a cooperative game is given by specifying a value for every coalition. Formally, the game (coalitional game) consists of a finite set of players N , called the grand coalition, and a characteristic function $v: 2^N \rightarrow R$ from the set of all possible coalitions of players to a set of payments that satisfies $v(\varphi) = 0$. The function describes how much collective payoff a set of players can gain by forming a coalition and the game is sometimes called a value game or a profit game. The players are assumed to choose which coalitions to form, according to their estimate of the way the payment will be divided among coalition members. Conversely, a cooperative game can also be defined with a characteristic cost function $c: 2^N \rightarrow R$ satisfying $c(\varphi) = 0$. In this setting, players must accomplish some task, and the characteristic function c represents the cost of a set of players accomplishing the task together. A game of this kind is known as a cost game [5]. For allocating profits, some concept's solutions refer to a 'range' of values that fulfill certain conditions like 'The Core' and some concept's solutions suggest a unique point like 'The Shapley value' and 'The Nucleolus'.

The core [4]: Let v be a game. The core of v is the set of payoff vectors

$$C(v) = \left\{ x \in R^N : \sum_{i \in N} x_i = v(N); \sum_{i \in S} x_i \geq v(s), \ \forall \ S \subseteq N \right\}. \tag{1}$$

In words, the core is the set of imputations under which no coalition has a value greater than the sum of its members' payoffs. Therefore, no coalition has incentive to leave the grand coalition and receive a larger payoff.

The Shapley value [13]: It is one way to distribute the total gains to the players, assuming that they all collaborate. It is a "fair" distribution in the sense that it is the only distribution with certain desirable properties listed below. According to the Shapley value, the amount that player i gets given a coalitional game (v, N) is

$$\phi_i(v) = \sum_{S \subseteq N \backslash \{i\}} \frac{|S|! \ (n - |S| - 1)!}{n!} [v(S \subseteq \{i\}) - v(S)] \tag{2}$$

where n is the total number of players and the sum extends over all subsets S of N not containing player i. The formula can be interpreted as follows: imagine the coalition being formed one actor at a time, with each actor demanding their contribution $v(S \subseteq \{i\}) - v(S)$ as a fair compensation, and then for each actor take the average of this contribution over the possible different permutations in which the coalition can be formed.

The nucleolus [12]: Let $v: 2^N \rightarrow R$ be a game, and let $x \in R^N$ be a payoff vector. The excess of x for a coalition $S \subseteq N$ is the quantity

$$v(S) - \sum_{i \in S} x_i \tag{3}$$

that is, the gain that players in coalition S can obtain if they withdraw from the grand coalition N under payoff x and instead take the payoff $v(S)$. Now let $\theta(x) \in R^{2^N}$ be the vector of excesses of x, arranged in non-increasing order. In other words, $\theta_i(x) \geq \theta_j(x), \ \forall \ i < j$. Notice that x is in the core of v if and only if it is a pre-imputation and $\theta_1(x) \leq 0$. To define the nucleolus, we consider the lexicographic ordering of vectors in R^{2^N}: For two payoff vectors x, y, we say $\theta(x)$ is lexicographically smaller than $\theta(y)$ if for some index k, we have $\theta_i(x) = \theta_j(y), \ \forall \ i < k$ and $\theta_k(x) = \theta_k(y)$. The nucleolus of v is the lexicographically minimal imputation, based on this ordering.

➢ 3. Overview of proposed approach and model

Conventional time-cost trade-off analysis adopts that the time and cost of an option within an activity are deterministic. But, in reality they are ambiguous. This aspect should therefore, be considered while analyzing the time cost trade-off problem, when minimizing project duration or cost. Simulation techniques are beneficial for analyzing stochastic effects, but a general strategy/ algorithm is required to monitor the analysis to obtain optimal solutions. This paper presents an extended approach which combines simulation techniques and genetic algorithms to solve the time-cost trade-off problem under uncertain situations. The results show that genetic algorithms can be integrated with simulation techniques to provide an efficient and practical means of obtaining optimal project schedules while assessing the associated risks in terms of time and cost of process startup activities. This approach provides planning engineers of any polyester company with a new way of analyzing process startup activities time/cost decisions in a more realistic manner. Historical time/cost data and available options to complete process startup activities can be modeled, so that planning engineers can identify the best strategies to take to complete the project at minimum time and cost. Also, what-if scenarios can be explored to decide the desired/optimal time and/or cost in planning and executing process startup activities.

Time, along with the cost and quality, is one of the most crucial factors in successful process startup activities. An important challenge in project scheduling is workmen efficiency in all activities. Obviously, each planning engineers tends to have more time and efficiency in his activities to more reliably decrease their costs. Many factors like; weather condition, material, equipment's or labor shortage, or price fluctuations, inclination to completion of activities in shorter time to catch the acceleration award or to dispatch the equipment's to other sites and extra can affect time and total cost of the project. These factors make some periods of time more valuable for planning engineers because of higher efficiencies and lower costs in these periods. In lower efficiency, cost for workmen increases for execution of a specified volume of anticipated work. This worthy time period differs from case to case based on specialty of workmen. In sequential projects, it is possible for subsequent total employee cum casual labours to trade time and decrease their total cost. This trading scheme can be performed according to time-cost and time-efficiency functions of total employees and finally may lead to an efficient schedule during shutdown. Based on information and experiences that planning manager has, he can specify his time-cost and time- efficiency functions individually. Hereby, a negotiation space can be developed between employee including casual labours at the beginning of the shutdown activities. The negotiation space in such problems presents the coalition space in cooperative game theory. For systematic and fair allocation of benefits resulted from cooperation, individual and group rationality are considered to make employees come into cooperation in more benefit. In Coalition space, the players decide to optimize benefits or cost of coalition without any restriction based on primary contracts. At the end of negotiation process, if the players come to new agreements, new contracts are signed between them and the excess benefits due to cooperation are allocated fairly for workmen.

➢ 4. The mathematical model

Initially the objective function is formulated based on time-cost relation of the activities for each major departments which is going to contribute during process startup activities, then followed by time-efficiency function to each activity. This function specifies daily, weekly or monthly work efficiency in any period of time for each workman who includes company employee and each casual worker supplied by different subcontractors under consideration. In a scientific definition, efficiency is the ratio of the work done by a work group to the work that can be done potentially by the same group. According to this definition, the efficiency always has a value between 0 and 1. It is necessary to consider the effect of anticipated efficiency throughout the project for estimation of total cost to company. In other words, excess cost due to efficiency decline in some periods of time, should be considered in modeling process. Labour cost is the payoff function of workmen including both permanent company employee and contractual worker.

From the total workmen, consider R_{ij}^{r} is the number of workmen type *j* required for activity *i* on time *t*, R_{ij}^{n} is the number of workmen type *j* required for activity *i* on time *t* available at normal condition, M_{ij} is the daily cost to company (CTC) of workmen type *j* required for activity *i*, P_{ij} is the daily cost to company (CTC) of workmen type *j* required for activity *i* above normal condition and t_i the time activity *i*. Thus,

$$C_{d_{ij}} = R_{ij}^{r} \times M_{ij} \times t_i + P_{ij} \times t_i \times (R_{ij}^{r} - R_{ij}^{n}) \tag{4}$$

where the premium is added when number of workmen of type j required for activity *i* exceeds of availability, so that P_{ij} is given by

$$P_{ij} = \begin{cases} V_{ij} - M_{ij} & \text{for } R_{ij}^{r} > R_{ij}^{n} \\ 0 & \text{for } R_{ij}^{r} \le R_{ij}^{n} \end{cases} \tag{5}$$

where V_{ij} is the cost rates of workmen type *j* at activity *i* above normal process startup availability limit.

It is proposed daily average cost and daily real cost is calculated:

$$c' = C_{d_{ij}}(T)/T \tag{6}$$

$$c(j) = c'/R(j) \tag{7}$$

where

c' denotes the daily average cost to company for workmen

$C(T)$ is the cost of process startup activities in time T

T is the time of project (day) for workmen

$c(j)$ is the daily real cost for workmen

j is the index of day

$R(j)$ represents the efficiency in day of j.

Now real cost of project can be calculated considering time-cost and time-efficiency functions by integrating daily real costs [1]:

$$C_R(T) = \sum_{S}^{F} c(j) = C(T)/T \sum_{S}^{F} 1/R_i(j) \qquad (8)$$

$$T = F - S \qquad (9)$$

where:

$C_R(T)$ denotes the real cost of all activities for a process startup activities
S is the start date of shutdown activities.
F represents finish date of startup activities.

In coalition status, subsequent workmen can trade time and decrease their total real cost. The model allows the subsequent planning engineers to choose the best start and finish date ignoring process startup constraints. Decision variable of the model is start and finish time of each process startup activities in coalition [1].

$$C_R(S) = Min \sum_{i \in S} \left\{ C_i(T_i)\ (1/T_i) \sum_{S_i}^{F_i} 1/R_i(j) \right\} \qquad (10)$$

$$T_i = F_i - S_i \quad \forall\, i \in S \qquad (11)$$

$$F_i \le S_{i+1} \quad \forall\, i \in S \qquad (12)$$

To solve simple problems, it is possible to use total search method while in complicated problems, the metaheuristic methods may be used. After time trading between process startup activities, it is necessary to assign new process startup activities have to accept all risks of process startup activities.

➢ 5. Illustrative Example

An illustrative example is presented to illustrate the process and performance of the proposal approach. Time-cost and Time-efficiency functions for activities are represented in table 1, 2, 3 & 4.

The model is solved in two cases:
1- Ignoring time-efficiency function
2- Considering time-efficiency function

Using equations (10), (11) and (12), total real costs are calculated for all coalitions formed by departmental workmen {1}, {2} and {3} that are represented in table 5 and 6.

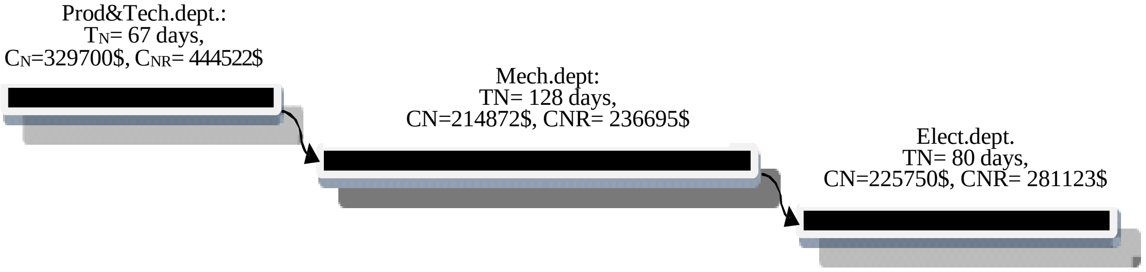

Figure 1: workmen position in process startup activities

where T_N = normal time for process startup sub-activities, C_N = cost in normal time without considering efficiency, C_{NR} = real cost in normal time.

Time	60	61	62	63	66	67	68	74	77	78	84	87
Cost	364350	363300	359100	341250	339150	329700	320250	313950	312900	307650	301350	300300

Table 1: Time-cost function for Production & Technical department {1}

Time	100	101	102	100	104	105	106	107	108	109	110	111
Cost	279972	269472	268947	268422	252672	252147	251622	249417	246267	231042	223167	222642
	112	114	115	116	118	119	120	121	122	124	125	126
	222117	221067	220542	220017	219387	218862	218337	218022	217497	216447	215922	215397
	128	131	132	133	134	137	138	139	140	142	143	145
	214872	214557	214137	213822	213297	213171	213087	212457	212037	211827	211617	211197
	148	151	154	156	158	159	161	169				
	210567	210147	210021	209895	209790	209727	209622	209454				

Table 2: Time-cost function for Mechanical department {2}

Time	60	61	62	63	65	66	67	68	71	73	74	77
Cost	279972	269472	268947	268422	252672	252147	251622	249417	246267	231042	223167	222642
	78	80	81	83	84	87	90	92	94	102	105	
	214872	214557	214137	213822	213297	213171	213087	212457	212037	211827	211617	

Table 3: Time-cost function for Electrical department {3}

Date (days)	0-10	11-20	21-30	31-50	51-70	71-100	101-195	195-254	255-275
Prod&Tech.dept.	0.5	0.5	0.75	1	1	1	1	-	-
Mech.dept.	-	-	-	-	0.5	0.75	1	1	-
Elect.dept.	-	-	-	-	-	-	1	1	0.5

Table 4: Time-efficiency function for departments in each period of project duration

Coalition	{1}	{2}	{3}	{1,2}	{1,3}	{2,3}	{1,2,3}
Cost	444522	236695	280741	523992	555450	425922	739242
Start-Finish	0-67	67-195	195-273	0-84, 84-195	0-67, 195-275	67-182, 82-275	0-78, 78-189, 189-275
Duration	67	128	78	84,111	67,80	115,93	78,111,86

Table 5: Cost and duration for each coalition not considering time-efficiency function

Coalition	{1}	{2}	{3}	{1,2}	{1,3}	{2,3}	{1,2,3}
Cost	444522	236695	280741	583388	725262	478099	859146
Start-Finish	0-67	67-195	195-273	17-85, 85-195	0-67, 195-273	67-177, 177-260	19-87,87-197, 197-275
Duration	67	128	78	68,110	67,78	110,83	68,110,78

Table 6: Cost and duration for each coalition considering time-efficiency function

For coalition {1,2,3} and in the case of ignoring time-efficiency function, total real cost is decreased from 770322$ to 739242$ (Table 5). This saving can be allocated to the workmen in the coalition according to one of available approaches (i.e. the shapely value, the nucleolus and etc.). Considering time- efficiency, it is evident that the entire process startup time is not necessarily utilized. For example, Prod&Tech.dept.{1} initiates his process startup activities on day 19 where in previous days thus had anticipated very low efficiency. Using various solution concepts in cooperative game theory, the total real cost of grand coalition, {1,2,3}, could be allocated between its players. The results are indicated in tables 7 and 8.

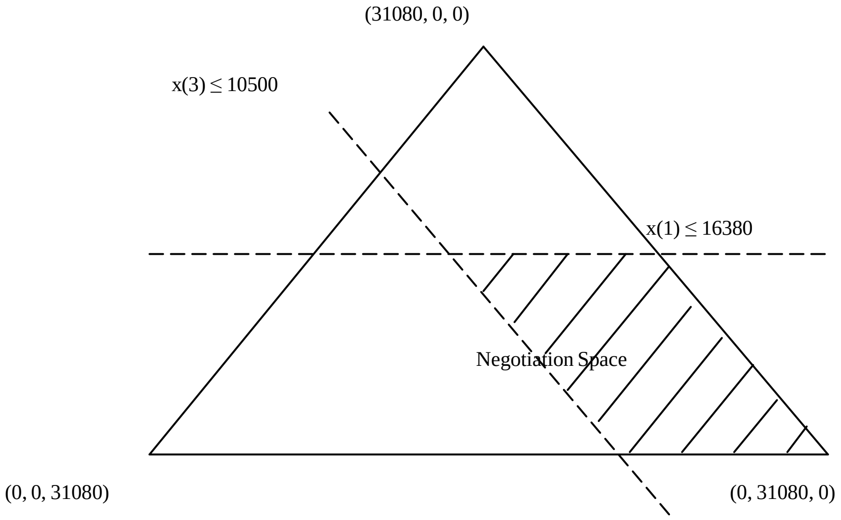

Figure 2: Space of the core not considering time-efficiency function

	Prod&Tech.dept.	Mech.dept.	Elect.dept.
Cost allocated by The Shapley value	320811	198633	219801
Benefit by The Shapley value	8889	16239	5949
Benefit (%) by The Shapley value	2.70%	7.56%	2.64%
Cost allocated by The Nucleolus	324450	194292	220500
Benefit by The Nucleolus	5250	20580	5250
Benefit (%) by The Nucleolus	1.59%	9.58%	2.33%

Table 7. Cost allocation not considering time-efficiency function

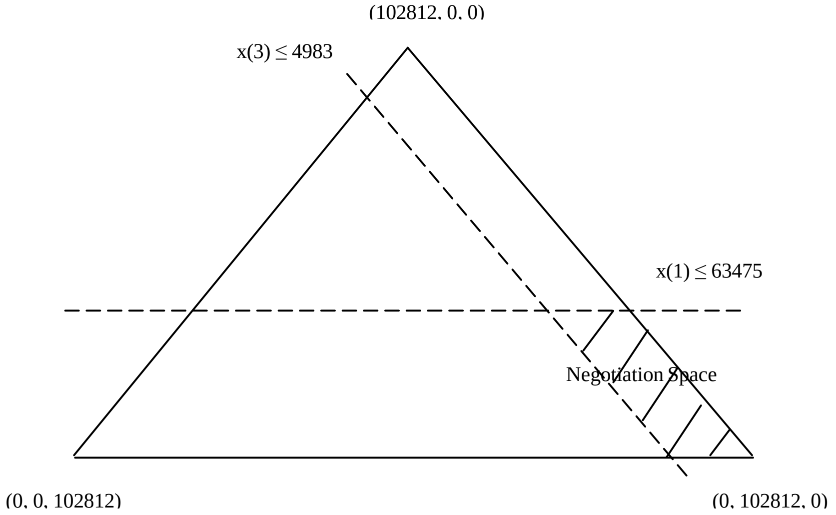

Figure 3: Space of the core in case of considering time-efficiency function

	Prod&Tech.dept.	Mech.dept.	Elect.dept.
Cost allocated by The Shapley value	407060	179565	272523
Benefit by The Shapley value	37464	57133	8217
Benefit (%) by The Shapley value	8.43%	24.14%	2.93%
Cost allocated by The Nucleolus	442031	138867	278250
Benefit by The Nucleolus	2493	97829	2493
Benefit (%) by The Nucleolus	0.56%	41.33%	0.89%

Table 8. Cost allocation considering time-efficiency function

Because of Mech.dept.{2} key role in all beneficial coalitions, he should earn more benefit than others. Therefore all solution concepts have assigned the most shares to Mech.dept.{2}. Additionally, this game for Prod&Tech.dept.{1} is more beneficial than {3} because coalition {1, 2} is more beneficial than coalition {1, 3}.

➢ 6. Summary, conclusions and additional work

Most common process startup activities include many sequential activities which are implanted by different departments but for the sake of brevity we have considered only three major departments. The allocated time to workmen according to planned activities cannot be the optimal time because of some constrains which workmen are involved in. These restrictions affect work efficiency and real cost of process startup activities. In coalition status, process startup activities may estimate optimal time that consequently decreases total real cost. Results show that ignoring time-efficiency function, process startup activities duration is decision variable of model and entire process startup activities time is used by workmen. On the other hand, considering time-efficiency function, start and finish time of process startup activities affect total real cost. By distributing benefits of coalition fairly, all workmen have a good reason to contribute.

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